PHYSICS BOOK

P.B 1st year

2025

PHSICS

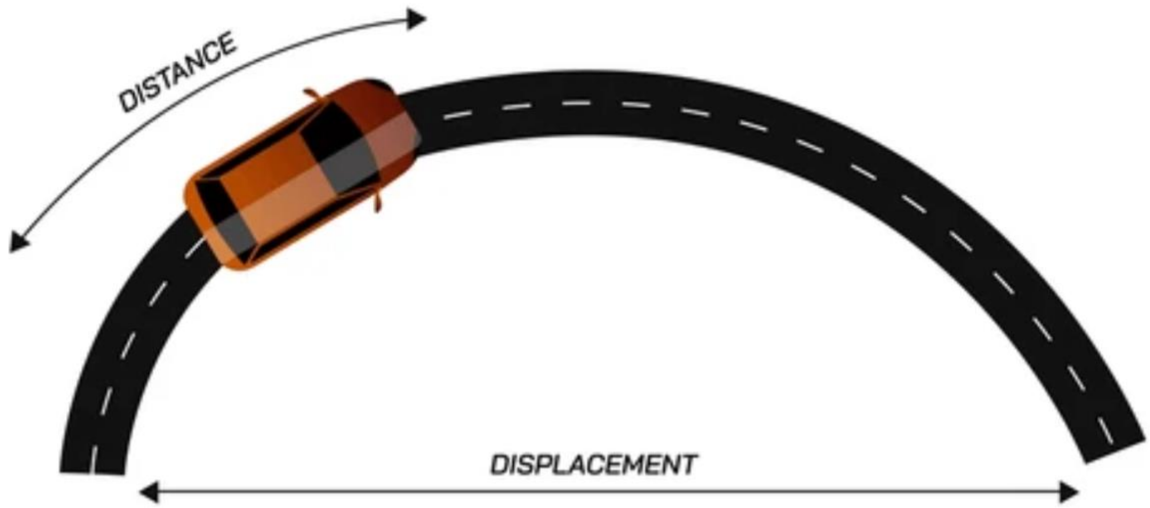
NOTES GRADE-I

Contents

CHAPTER

02

VECTORS



Learning Objectives

Scalars

Vectors

Graphical Representation of a Vector

Rectangular Components of a Vector

Determination of a Vector from its Rectangular Components

Product of two Vectors

Scalar or Dot Product

Characteristics of Scalar Product

Vector or Cross Product

Characteristics of Cross Product

Examples of Vector Product

BASIC CONCEPT OF SCALARS AND VECTORS

Scalars and vectors are basic concepts in physics. Many problems in physics require to distinguish between scalar and vector quantities to apply the correct mathematical and conceptual approaches. Understanding scalars and vectors help us to grasp how physics applies to real-world situations, such as calculating the total distance traveled(scalar) or determining the magnitude and direction of force (vector). Learning these concepts develops critical thinking and problem-solving skills. This chapter is primarily concerned with vector algebra and its application .

2.1 SCALARS

Scalars are physical quantities that are described solely by a magnitude (Size or amount) without any mention of direction. Thus, scalars are directionless and can be fully characterized by a single number and its associated unit.

Examples:
Mass: The amount of matter in an object. For example, 2 kg.
Distance: The total length of path travelled by an object irrespective of the direction. For example, 50m.
Speed: The rate at which an object covers distance. For example, 40 kmh^{-1}.
Time: The ongoing sequence of events taking place. For example, 20 seconds.
Energy: The capacity to do work. For example, 25 J.
Temperature: A measure of the average kinetic energy of particles in a substance. For example, 20°C.

2.2 VECTORS

Those physical quantities which have length (magnitude or size) as well as direction in which they act.
Examples:
Displacement: The change in position of an object. It has length, a distance (magnitude) and a direction (e.g. 10 m towards west).
Velocity: The speed of an object in a particular direction (e.g. 50 km h^{-1} towards west).
Acceleration: The rate of change of velocity, which includes changes in speed and or direction (e.g. 10 ms^{-2} upward).
Force: A push or pull acting on an object, determined by its magnitude and direction (e.g. 20 N to the right)

Vector Representation

There are two ways to represent a vector quantity.

(a) Symbolic representation
(b) Graphical representation

(a) Symbolic Representation

In books, vectors are usually denoted by bold face characters such as **A, d, r** and **v** while in handwriting, we put an **arrowhead** over the letter e.g. $\vec{d}$. If we wish to refer only to the magnitude of a vector $\vec{d}$ we use light face type such as d or $|\vec{d}|$.

(b) Graphical Representation

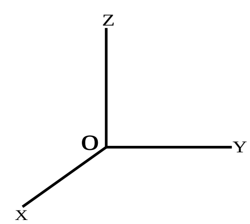
- A vector is represented graphically by a directed line segment with an arrow head.
- The length of the line segment, according to a chosen scale, corresponds to the magnitude of the vector and an arrow head at one end gives the direction of the vector.

Example

The vector shown in the diagram can be written as

vector $\vec{A}$, and its magnitude is written as $|\vec{A}|$.

(ii) Rectangular Coordinate System



The system of perpendicular coordinate axes is known as ***Cartesian*** or ***rectangular coordinate system***.

The rectangular coordinate system consists of **two** axes in plane while **three** axes in space.

Coordinate Axes

Two or three reference lines drawn at right angles to each other are called *coordinate axes* or *reference axes*.

Origin: Point of intersection of these reference lines is called *origin*.

x-axis: Usually, Horizontal line is named as *x-axis*, with the positive direction to the right.

y-axis: Usually, vertical line is named as *y-axis*, with the positive direction upward.

z-axis: The third mutually perpendicular axis is named as *z-axis*.

Rectangular Components of a Vector

A component of a vector is its effective value in a given direction. A vector may be considered as the resultant of its component vectors along the specified directions. It is usually convenient to resolve a vector into its components along the mutually perpendicular directions. Such components are called rectangular components.

Let there be a vector **A** represented by a line OP making an angle θ with the x-axis. Draw projection OM of vector **A** on x-axis and projection ON of vector **A** on y-axis as shown in Fig.2.1. Projection OM being along x-direction represented by $A_x\hat{i}$ and projection ON

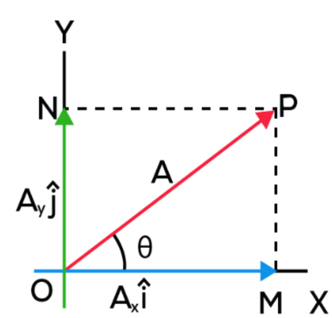


Fig. 2.1

along y-direction is represented by $A_y\hat{j}$. By applying head to tail rule:

$$A = A_x\hat{i} + A_y\hat{j} \text{ ----------------- (2.1)}$$

Thus, $A_x\hat{i}$ and $A_y\hat{j}$ are the components of vector **A**. Since these are at right angle to each other, they are called rectangular components of **A**. Considering the right angled triangle OMP, the magnitude of $A_x\hat{i}$ or x-component of **A** is:

$$A_x = A\cos\theta \text{ ------------------------ (2.2)}$$

And the magnitude of $A_y\hat{j}$ or y-component of **A is**

$$A_y = A\sin\theta \text{ ------------------------ (2.3)}$$

Determination of a Vector from its Rectangular Components

If the rectangular components of a vector as shown in Fig (2.1) are given, we can find out the magnitude of the vector by using Pythagorean Theorem

In the right angle Δ OMP

$$(OP)^2 = (OM)^2 + (MP)^2$$

or

$$A^2 = A_x^2 + A_y^2 \text{ ------------------ (2.4)}$$

$$A = \sqrt{A_x^2 + A_y^2}$$

The direction θ is given by

$$\tan\theta = \frac{MP}{OM} = \frac{A_y}{A_y}$$

or

$$\theta = \tan^{-1}\left(\frac{A_y}{A_y}\right) \text{ -----------------(2.5)}$$

Example 2.1: Find the angle between two forces of equal magnitude when the magnitude of their resultant is also equal to the magnitude of either of these forces.

Solution: Let θ be the angle between two forces F_1 and F_2 where F_1 is along x-axis. Then x-component of their resultant will be:

$$R_x = F_1\cos\theta + F_2\cos\theta$$

$$R_x = F_1 + F_2\cos\theta$$

And y-component of their resultant is

$$R_y = F_1\sin\theta + F_2\sin\theta$$

$$R_y = F_2\sin\theta$$

The resultant is given by $R^2 = R_x^2 + R_y^2$

As $R = F_1 = F_2 = F$

Hence $F^2 = (F + F\cos\theta)^2 + (F\sin\theta)^2$

$$F^2 = F^2 + F^2\cos^2\theta + 2F^2\cos\theta + F^2\sin^2\theta$$

Or $0 = 2F^2\cos\theta + F^2(\cos^2\theta + \sin^2\theta)$

Or $0 = 2F^2\cos\theta + F^2$

Or $\cos\theta = -0.5$

Or $\theta = \cos^{-1}(-0.5) = 120^o$

Example 2.2: Fatima is pulling her trolley beg while climbing up the ramp at her school gate. Find the force with which she is pulling her bag, if x-component and y-component of her force are 12 N and 5 N, respectively.

Given

$F_x = 12\text{ N}$

$F_y = 5\text{ N}$

To Find

(a) $F = |\vec{F}| = ?$

(b) $\theta = ?$

Solution

(a) As $F = \sqrt{F_x^2 + F_y^2}$

Putting the values, we have

$$F = \sqrt{(12)^2 + (5)^2} = \sqrt{144 + 25}$$

or $F = \sqrt{169}$

Hence **F = 13 N**

(b) Angle formed by bag

$$\theta = \tan^{-1}\frac{F_y}{F_x} = \tan^{-1}\frac{5\text{ N}}{12\text{ N}}$$

or $\theta = \tan^{-1} = 0.42$

Hence **θ = 22.78°**

2.3 PRODUCT OF TWO VECTORS

There are two types of vector multiplications. The product of these two types are known as scalar product and vector product As the name implies, scalar product of two vector quantities is a scalar quantity, while vector product of two vector quantities is a vector.

Scalar or Dot Product

If product of two vectors is a scalar quantity, then the product is called scalar or dot product. If $\vec{A}$ and $\vec{B}$ are two non-zero vectors then their dot product is defined as:

$$\vec{A}.\vec{B} = AB\cos\theta \text{ ---------------- (2.6)}$$

Where A and B are the magnitudes of **A** and **B** respectively. θ is angle between **A** and **B**.

For physical interpretation of dot product of two vectors **A** and **B**, these are first brought to common origin (Fig.2.2 a), then

A . B= A (projection of **B** on **A**)

Or

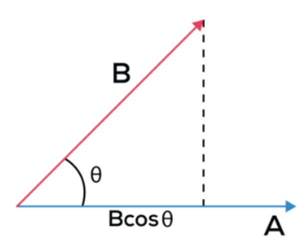


Fig. 2.2 (a)

A . B = A (magnitude of component of **B** in the direction of **A**)
A . B = A (B cosθ) = AB cosθ
From Fig (2.2 b),
B . A = B (projection of **A** on **B**)
Or

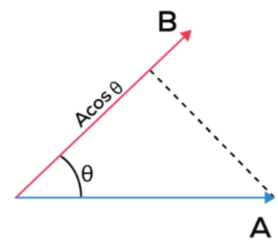


Fig. 2.2 (b)

B . A = B (magnitude of component of **A** in the direction of **B**)
B . A = B (A cosθ)
B . A = BA cosθ

Example:

We come across this type of product when we consider the work done by a force F whose point of application moves a distance d in a direction making an angle θ with the line of action of F, as shown in Fig. 2.3.

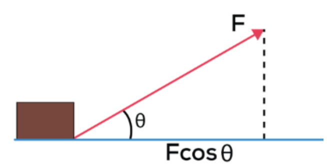


Fig. 2.3

Work done = (effective component of force in the direction of motion) × distance moved

Work done = (F cos θ) d= Fd cos θ

Using vector notation

F . d = Fd cos θ = Work done

Characteristics of Scalar Product

1. Since **A . B** = AB cosθ and **B . A** = BA cosθ, hence **A . B** = **B . A.** The order of multiplication is irrelevant. In other words, scalar product is commutative.
2. Scalar product of two mutually perpendicular vectors is zero.

   A . B = AB cos90° = 0

   In case of unit vectors $\hat{i}$, $\hat{j}$ and k, since they are mutually perpendicular , therefore,

$$\hat{i}\cdot\hat{j} = \hat{j}\cdot k \quad k\cdot\hat{i} = 0 \text{----------------} (2.7)$$

3. The scalar product of two parallel vectors is equal to product of their magnitudes. Thus, for parallel vectors (θ = 0°).

   A . B = AB cos0° = AB

   In case of unit vectors

$$\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} \quad k\cdot k = 1 \text{----------------} (2.8)$$

And for anti parallel vectors (θ = 180°)

A . B = AB cos180° = - AB

4. Self-dot product of a vector **A** is equal to the square of its magnitude.

$$\mathbf{A}\,.\,\mathbf{A} = AA\cos 0 = A^2$$

5. Scalar product of two vectors **A** and **B** in terms of rectangular components

$$\mathbf{A}\,.\,\mathbf{B} = \left(A_x\hat{i}+A_y\hat{j}+A_z\hat{k}\right).\left(B_x\hat{i}+B_y\hat{j}+B_z\hat{k}\right)$$

Or $\mathbf{A}\,.\,\mathbf{B} = A_xB_x + A_yB_y + A_zB_z$ ---------------- (2.9)

Equation (2.9) can be used to find the angle between two vectors. Since ,

$$\mathbf{A}\,.\,\mathbf{B} = AB\cos\theta = A_xB_x + A_yB_y + A_zB_z$$

$$\cos\theta = \frac{A_xB_x + A_yB_y + A_zB_z}{AB}$$

$$\theta = \cos^{-1}\left(\frac{A_xB_x + A_yB_y + A_zB_z}{AB}\right)$$ ------------------ (2.10)

Example 2.3: A force $\vec{F} = 2\hat{i} + 3\hat{j}$ units, has its point of application moved from point A (1,3) to the point B (5,7). Find the work done.

Solution: Position vector of point $A = \vec{r}_A = \hat{i} + 3\hat{j}$ and Position vector of point $A = \vec{r}_B = 5\hat{i} + 3\hat{j}$

Displacement = d = $\vec{r}_B - \vec{r}_A = (5\hat{i} + 7\hat{j}) - (\hat{i} + 3\hat{j}) = 4\hat{i} + 4\hat{j}$

$$W = \vec{F}.\vec{d} = 8 + 12 = 20 \text{ units}$$

Example 2.4: Find the projection of vector $\vec{A} = 2\hat{i} - 8\hat{j} + \hat{k}$ in the direction of the vector $\vec{B} = 3\hat{i} - 4\hat{j} - 12\hat{k}$.

Solution

Given Data

$\vec{A} = 2\hat{i} - 8\hat{j} + \hat{k}$

$\vec{B} = 3\hat{i} - 4\hat{j} - 12\hat{k}$

To Find

The projection of **A** on **B =** $A\cos\theta$ = ?

Calculation

If θ is the angle between $\vec{A}$ and $\vec{B}$, then A $\cos\theta$ is the required projection.

By definition $\vec{A}\,.\,\vec{B} = AB\cos\theta$

$A\cos\theta = \dfrac{\vec{A}.\vec{B}}{B} = \vec{A}.\hat{B}$

Where $\hat{B}$ is the unit vector in the direction of $\vec{B}$

$B = \sqrt{3^2 + (-4)^2 + (-12)^2} = 13$

Now

$\hat{B} = \dfrac{\vec{B}}{B}$

Therefore, $\hat{B} = \dfrac{3\hat{i} - 4\hat{j} - 12k}{13}$

The projection of $\vec{A}$ on $\vec{B}$ = $(2\hat{i} - 8\hat{j} + k).\frac{3\hat{i} - 4\hat{j} - 12k}{13}$

$$= \frac{(2)\ (3)\ +\ (-8)\ (-4)\ +\ 1\ (-12)}{13} = \frac{26}{13} = 2$$

Vector Or Cross Product

The vector product of two vectors A and B is a vector which is defined as

$$\mathbf{A \times B} = AB \sin\theta\ \hat{n} \text{ ---------------------- (2.11)}$$

where **n** is a unit vector perpendicular to the plane containing **A** and **B** as shown in Fig. (2.4. a). Its direction can be determined by right hand rule. For that purpose, place together the tail of vectors **A** and **B** to define the plane of vectors **A** and **B**. The direction of the product vector is perpendicular to this plane. Rotate the First vector **A** into **B** through the smaller of the two possible angles and curl the fingers of the right hand in the direction of rotation, keeping the thumb erect. The direction of the product vector will be along the erect thumb, as shown in the Fig (2.4.b). Because of this direction rule, **B x A** is a vector opposite in sign to **A x B** (Fig. 2.4. c). Hence,

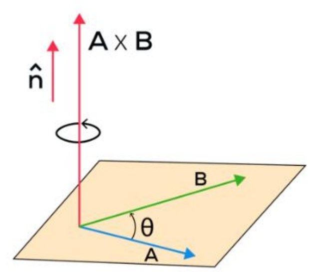


Fig. 2.4 (a)

$$\mathbf{A \times B = - B \times A} \text{ -----------------------(2.12)}$$

Characteristics of Vector or Cross Product

1. Since **A** × **B** is not same as **B** × **A**, the cross product is non commutative, as shown in Fig (2.4 a) and Fig (2.4 c)

   A × **B** ≠ **B** × **A** but **A** × **B** = **- B** × **A**

2. The cross product of two perpendicular vectors has maximum magnitude **A x B** = ABsin90º $\hat{n}$ = AB $\hat{n}$

   In case of unit vectors, since they form a right handed system and are mutually perpendicular. So,

   $$\hat{i}\times\hat{j}=\hat{k} \quad \hat{j}\times\hat{k}=\hat{i} \quad \hat{k}\times\hat{i}=\hat{j}$$

3. The cross product of two parallel vectors is null vector, because for such vectors θ = 0º or 180°, Hence,

   A x B=ABsin0°$\hat{n}$ = ABsin 180°$\hat{n}$ = **O**

   As consequence **A x A** = **O**

Also $\hat{i}\times\hat{i}=\hat{j}\times\hat{j}=\hat{k}\times\hat{k}=\vec{0}$ ------------- (2.13)

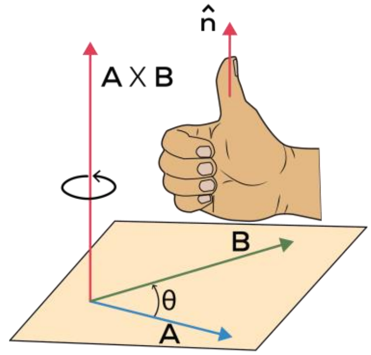


Fig. 2.4 (b)

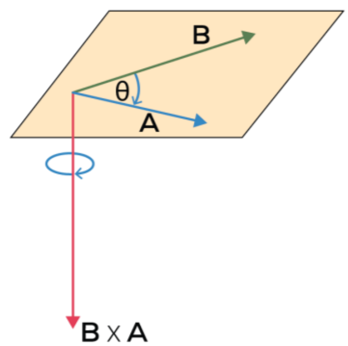


Fig. 2.4 (c)

4. Cross product of two vectors **A** and **B** in terms of rectangular components is

   A = $A_x\hat{i}+A_y\hat{j}+A_z\hat{k}$ and **B** = $B_x\hat{i}+B_y\hat{j}+B_z\hat{k}$

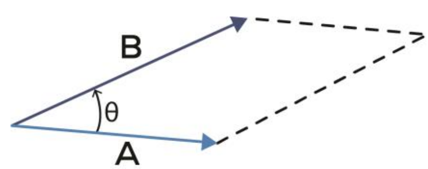
   A x B $=\left(A_x\hat{i}+A_y\hat{j}+A_z\hat{k}\right)\times\left(B_x\hat{i}+B_y\hat{j}+B_z\hat{k}\right)$

   $$\vec{A}\times\vec{B}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ A_x & A_y & A_z\\ A_y & B_y & B_z\end{vmatrix}$$

   $$\vec{A}\times\vec{B}=\begin{vmatrix}A_y & A_z\\ B_y & B_z\end{vmatrix}\hat{i}-\begin{vmatrix}A_x & A_z\\ B_x & B_z\end{vmatrix}\hat{j}+\begin{vmatrix}A_x & A_y\\ B_x & B_y\end{vmatrix}\hat{k}$$

   $$\vec{A}\times\vec{B}=\left(A_yB_z-A_zB_y\right)\hat{i}-\left(A_xB_z-A_zB_x\right)\hat{j}+\left(A_xB_y-A_yB_x\right)\hat{k}$$

   $$\vec{A}\times\vec{B}=\left(A_yB_z-A_zB_y\right)\hat{i}+\left(A_zB_x-A_xB_z\right)\hat{j}+\left(A_xB_y-A_yB_x\right)\hat{k}$$



5. The magnitude of **A x B** is equal to the area of parallelogram formed with A and B as two adjacent sides in above figure.

Examples of vector Product

i. When a force **F** is applied on a rigid body at a point whose position vector is r from any point on the axis about which the body rotates, then the turning effect of the force called the torque is given by the vector product of r and F.

T = r x F

ii. The force on a particle of charge q and velocity **v** in a magnetic field of strength **B** is given by vector product.

$$\mathbf{F} = q\,(\mathbf{v} \times \mathbf{B})$$

Example 2.5: Find the area of the parallelogram whose adjacent sides are given by the vectors: $\mathbf{A} = \hat{i} + 6\hat{j} + 2\hat{k}$ m and $\mathbf{B} = 7\hat{i} + \hat{j} + 5\hat{k}$ m.

Given

$A = \hat{i} + 6\hat{j} + 2\hat{k}$ m $\qquad$ $B = 7\hat{i} + \hat{j}\ 5\hat{k}$ m

To Find

$\vec{A} \times \vec{B} = ?$

Solution

As we know that vector product gives us the area of parallelogram.

So, $$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 6 & 2 \\ 7 & 1 & 5 \end{vmatrix}$$

or $$\vec{A} \times \vec{B} = \hat{i}\begin{vmatrix} 6 & 2 \\ 1 & 5 \end{vmatrix} - \hat{j}\begin{vmatrix} 1 & 2 \\ 7 & 5 \end{vmatrix} + \hat{k}\begin{vmatrix} 1 & 6 \\ 7 & 1 \end{vmatrix}$$

Solving the above equation, we have

$$\vec{A} \times \vec{B} = (6\times5-2\times1)\hat{i} + (7\times2-1\times5)\hat{j} + (7\times6-1\times1)\hat{k}$$

or $$\vec{A} \times \vec{B} = (30-2)\hat{i} + (14-5)\hat{j} + (42-1)\hat{k}$$

Hence, $\boldsymbol{\vec{A} \times \vec{B} = 28\hat{i} + 9\hat{j} - 41\hat{k}\ m^2}$ **Ans.**

MULTIPLE CHOICE QUESTIONS

1. The angle at which dot product becomes equal to cross product:
(a) 65° (b) 45°
(c) 76° (d) 30°

2. The projectile gains its maximum height at an angle of:
(a) 0° (b) 45°
(c) 60° (d) 90°

3. The scalar product of two vector is maximum if they are:
(a) perpendicular (b) parallel
(c) at 30° (d) at 45°

4. It two components of a vector are equal in magnitude, the vector making angle with x-axis will be
(a) 30° (b) 60°
(c) 90° (d) 45°

5. The cross product of a $\vec{A}$ vector with itself is
(a) Zero (b) Null vector
(c) A^2 (d) A

6. $\vec{A}.\vec{B}=|\vec{A}\times\vec{B}|$ **if the angle between** $\vec{A}$ **and** $\vec{B}$ **is:**

(a) 0° (b) 90°
(c) 30° (d) 45°

7. $\hat{j}.(\hat{k}\times\hat{j})$ **is equal to**

(a) −1 (b) Zero
(c) 1 (d) 2

8. If $\vec{\mathbf{A}} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$ then magnitude of $\vec{\mathbf{A}}$

(a) $\sqrt{-3}$ (b) −1
(c) $\sqrt{29}$ (d) $\sqrt{-1}$

9. **Magnitude of resultant vector of 6N and 8N, which are perpendicular to each other is**

(a) 14 N (b) 10 N
(c) 20 N (d) 2 N

10. **The angle at which dot product is half of its maximum value is:**

(a) 0° (b) 30°
(c) 60° (D) 90°

11. $\vec{A}.\vec{B}=|\vec{A}\times\vec{B}|$ **if the angle between** $\vec{A}$ **and** $\vec{B}$ **is:**

(A) 0° (B) 90° (C) 30° (D) 45°

12. **Which of the following is equal to one?**

(A) $\hat{j}\times\hat{j}$ (B) $\hat{i}\times\hat{i}$ (C) $\hat{i}.\hat{i}$ (D) $\hat{i}.\hat{j}$

13. **Two forces of magnitude 10N each. Their resultant is equal to 20N. Then angle between them is:**

(A) 180° (B) 30° (C) 90° (D) 0°

14. **Unit vector of a given vector** $\overline{A}=4\hat{i}+3\hat{j}$ **is:**

(A) $\frac{4\hat{i}+3\hat{j}}{25}$ (B) 1

(C) $\frac{4\hat{i}+3\hat{j}}{5}$ (D) $\sqrt{\frac{4\hat{i}+3\hat{j}}{5}}$

15. **If** $\vec{\mathbf{A}} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$ **then magnitude of** $\vec{\mathbf{A}}$

(A) $\sqrt{-3}$ (B) −1

(C) $\sqrt{29}$ (D) $\sqrt{-1}$

SHORT QUESTIONS

1. Can scalar product of two vectors be negative? Given an example.
2. The scalar product of a vector A with an unknown vector B is zero. Assume that you are given a non-zero vector A. What can you conclude about B?

3. A force of 10 N makes an angle of 60° with x-axis. Find its x and y-components.
4. Suppose the sides of a closed polygon represent vector arranged head to tail rule. What is the sum of these vectors?
5. Can you add two scalars and two vectors using the same method? Explain.
6. How do you resolve a vector into its components?
7. Is it possible to add 5 in $\mathbf{2\hat{i}}$?
8. If all the components of the vector $\vec{A}_1$ and $\vec{B}_2$ were reversed, how would this alter $\vec{A}_1 \times \vec{B}_2$?
9. Make the three different conditions that could make $\vec{A}_1 \times \vec{A}_2 = \vec{0}$.
10. If one of the rectangular components of a vector is not zero, can its magnitude be zero? Explain.

NUMERICAL PROBLEMS

1. The magnitude of dot and cross products of two vectors are $6\sqrt{3}$ and 6 respectively. Find the angle between them. ($\theta = 30^{O}$ Ans)
2. If $\vec{\mathbf{A}} = \mathbf{4\hat{i} - 4\hat{j}}$, what is the orientation of $\vec{\mathbf{A}}$? ($\theta = 315°$ Ans)
3. Find the dot product of vectors **A = 3i + 4j – 2k** and **B = –1i + 2j + 5k**. (-5 Ans)
4. Two vectors **X =** $\hat{i} + 2\hat{j} + 2k$ and **Y =** $2\hat{i} - 2\hat{j} + k$ are perpendicular. Verify using the cross product magnitude. (9 Ans)
5. Vectors **U =** $3\hat{i} + \hat{j}$ and **V =** $2\hat{i} - 4\hat{j}$ form adjacent sides of a parallelogram. Find its area. (14 sq. units Ans)
6. Vectors **A**, **B** and **C** are 4 units north, 3 units west and 8 units east, respectively. Describe carefully (a) **A x B** (b) **A × C** (c) **B × C**.
7. Find the angle between the two vectors, $\vec{A} = 5\hat{i} + \hat{j}$ and $\vec{B} = 2\hat{i} + 4\hat{j}$ ($\theta = 37.4°$ Ans)
8. A force is acting on a body making an angle of 30° with the horizontal. The horizontal component of the force is 20 N. Find the force. (F = 23.1 N Ans)